Quick Challenge: 09-2013 , complex solutions of a non linear system

From brilliant.org:

Consider all ordered triples of complex numbers (a, b, c) which satisfy the following equations:

$$\begin{cases} ab + 3b = -9 \\ bc + 3c = -9 \\ ca + 3a = -9. \end{cases}$$

What is the largest possible value of $|abc|$?

Brilliant.org (as many other sites, but this one is *ad hoc*) provides a lot of smart and quick mathematical challenges, useful to learn or revise mathematical concepts and so forth. One can use it even to learn more about the **mathematical framework** of the HpS0g, that is very huge. This is the objective.

Note: we will try to avoid CAS manipulations as much as possible, plus we will not be so rigorous.

It is likely that there are many solutions for the problem, bu we see only some of them or just one.

Ideas

1. We have a nonlinear system with complex numbers. One tool, in the library of the hpS0g, that can handle that type of system is MSLV (multiple equations solver). But this tool is not so fast and a bit limited about non linear systems. So, searching for other programs, we can find one excellent substitution of MSLV (even if it comes from year 2000!), **SolveSys49** by *Sune Brendahl* on **hpcalc.org**. There can be also other better replacements to both MSL and SolveSys but actually we don't know them (if you find them, share your findings!).

Implementations

We just use the new solver library without programming. We recall Solvesys49, then we add the equations and we give to the solver an initial value for each unknown (called "guess"). Note that we want complex solutions so, unless the flag **-103** is set, we have to write initial values as complex numbers like $(5,0)$. Moreover, since the solver use

an heuristic (that is not perfect), for some initial values it ends in a blind alley, so don't get discouraged! So, after we give to the solver a good triple of initial values $(a_{0,}b_{0,}c_{0})$ we get the solution for each unknown. But we want $\max|a\cdot b\cdot c|$, how can we find the maximum? We can note that the nonlinear system, when numbers are decomposed in real and integer part, is composed by six equations with six unknowns $\{(a_{\Re},a_{\Im}),(b_{\Re},b_{\Im})(c_{\Re},c_{\Im})\}$ so it has only a solution. Thus we have to compute only one product, that between the result given by the solver, to get the maximum value of $|a\cdot b\cdot c|$.

Appendix

Tools used

- OpenOffice 4 (that is less painful than previous versions. To convert the text directly into a formula, just write the code for the formula, then select it and recall the formula options in the menu (I have assigned a shortcut to it). To change, definitely the font of a formula, change the font styles and the click "default")
- Offside as font. Cambria as font for formulas.
- Paint.net, to edit some images.
- Hp 50g graphing calculator to test the ideas.
- Debug4x to write RPL code for the 50g.

Versions

- 21.09.2013 first draft.